Logarithms

A $\log_a b = c \; means \; the \; same \; as \; a^c = b$

Eg Write down the value of $\log_2 32$.

Think about the power that 2 needs to be raised to so you have 32.

$\mathbf{2^5 = 32}$ so $\mathbf{\log_2 32 = 5}$

Write the following in log notation.

1 $2^3 = 8$
2 $10^{-3} = \frac{1}{1000}$
3 $2^{-3} = 0.125$
4 $64^{\frac{1}{3}} = 4$
5 $6^3 = 216$
6 $2^7 = 128$
7 $4^5 = 1024$
8 $9^3 = 729$
9 $x^3 = 50$
10 $6^x = 12$

Inverse functions for Logarithms

B $\boldsymbol{a^{\log_a x} = x}$ $\boldsymbol{\log_a(a^x) = x}$ $(for \; any \; positive \; a \neq 1)$

$\boldsymbol{e^{\ln x} = x}$ $\boldsymbol{\ln(e^x) = x}$

Remember that $\mathbf{\log_a a = 1}$ and $\mathbf{\ln e = 1}$

Eg Give exact values of x

$e^x = 20$
$\ln e^x = \ln 20$
$x = \ln 20$

$\ln x = 3$
$e^{\ln x} = e^3$
$x = e^3$

Solve these equations giving exact answers and a solution correct to 4 significant figures.

1 $e^x = 8$
2 $e^{3t} = 15$
3 $6e^x = 350$
4 $\ln x = 12$
5 $9 \ln x = 72$
6 $e^{4x} = 20$
7 $e^{5x} = 35$
8 $5 \ln 3 = 75$

For each part, use index notation to write f in terms of g.

1 $\log_a f = 2$ and $\log_a g = 6$
2 $\log_a 2f = 12$ and $\log_a g = 8$
3 $\log_a f = 5$ and $\log_{2a} g = 5$
4 $\log_a f = 12$ and $\log_{3a} 6g = 15$

Laws of Logarithms

C $\log_a x + \log_a y = \log_a(xy)$ $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$ $\log_a x^k = k\log_a x$

Eg Write the expression $\ln\frac{(ex)^3}{y^2}$ in terms of $\ln x$ and $\ln y$

$$\ln\frac{(ex)^3}{y^2} = \ln(ex)^3 - \ln y^2$$

$$= \ln e^3 + \ln x^3 - \ln y^2$$

$$= 3\ln e + 3\ln x - 2\ln y$$

$$= 3 + 3\ln x - 2\ln y$$

Use the laws of logarithms to simplify the following expressions as much as possible.

1 $3\log_a 4$
2 $\frac{1}{2}\log_g 16$
3 $\log 0.5$
4 $\log\sqrt[3]{7}$
5 $3 + \log_a 8$
6 $\frac{2}{3}\ln 343 - 2\ln 14$
7 $\log_a a^7$
8 Find the value of $6 + \log_a\frac{1}{a^2} + \log_c\sqrt{c}$

Changing the Base of a Logarithm

D $\log_a x = \frac{\log_{10} x}{\log_{10} a}$ $\log_a x = \frac{\ln x}{\ln a}$ $\log_a x = \frac{\log_b x}{\log_b a}$

Eg Find the value of $\log_8 9$

$$\log_8 9 = \frac{\log_{10} 9}{\log_{10} 8} = 1.056641667$$

For each of the following questions, write down the log in terms of $\log_{10}$, find the value to 5 s.f.

1 $\log_7 11$
2 $\log_{11} 7$
3 $\log_3 5$
4 $\log_8 5$
5 $\log_6 25$
6 $\log_9 99$

Write the following in terms of a single log.

1 $\frac{\log_3 2}{\log_3 3}$
2 $\frac{\log_7 6}{\log_8 8}$
3 $\log_5 3 + \log_4 7$

Solving Equations

E $\boldsymbol{a^{\log_a b} = b = \log_a a^b}$

Eg1 Solve $5^{2x} = 8$ to 3 sig. fig.

$\log 5^{2x} = \log 8$

$2x \log 5 = \log 8$

$2x = \frac{\log 8}{\log 5}$

$x = \frac{\log 8}{2 \log 5}$

$x = 0.7576014525$

$x \approx 0.758$

Solve the following to 3 significant figures.

1 $8^{3x} = 24$
2 $9^{x} = 35$
3 $7^{9x} = 56$
4 $16^{5x} = 82$
5 $25^{5x} = 360$

Eg2 Solve $9 \log_{10} x = 23$ to 3 d.p.

$\log_{10} x = \frac{23}{9}$

$10^{\log_{10} x} = 10^{\frac{23}{9}}$

$x = 10^{\frac{23}{9}}$

$x = 359.3813664$

$x \approx 359.381$

Solve the questions below to 4 significant figures.

6 $5 \log_{10} x = 55$
7 $9 \log_{10} x = 36$
8 $8 \log_{10} x = 12$
9 $3 \log_{10} x = 26$
10 $7 \log_{10} x = 19$
11 $3 \log_{10} x = 55$
12 $11 \log_{10} x = 77$
13 $12 \log_{10} x = 45$
14 $17 \log_{10} x = 69$
15 $10 \log_{10} x = 85$

Eg 3 Solve $\ln(3x-1)=8$ giving your answer correct to 6 significant figures.

$\ln(3x-1)=8$

$e^{\ln(3x-1)}=e^{8}$

$3x-1=e^{8}$

$x=\frac{e^{8}+1}{3}$

$x=$993.9859957

$x=993.986$

Eg 4 Find two solutions of $e^{x}+10e^{-1}=7$ giving your solution to 4 decimal places.

$e^{x}+10e^{-1}=7$

$e^{2x}+10e^{0}=7e^{x}$ (multiply throughout by e^{x} to get rid of e^{-1})

$e^{2x}-7e^{x}+10=0$

$y^{2}-7y+10=0$ (Substitute $y=e^{x}$)

$(y-2)(y-5)=0$

$y=2\ and\ y=5$

$e^{x}=2\ and\ e^{x}=5$

$\ln e^{x}=\ln 2\ \ and \ln e^{x}=\ln 5$

$\ln 2=0.6931471806\ and \ln 5=1.609437912$

$x=0.6931\ or\ x=1.6094$ to 4 decimal places.

16 $5e^{3t}=12$

17 $\ln(5x-3)=9$

18 $e^{x}+15e^{-1}=8$

19 $6^{50-x}=2^{50}$

20 $4^{x+1}=3^{2x}$

21 $\ln 5+\ln x=8$

22 $4^{2x}-17(4^{x})=-16$

23 $e^{4x}+4e^{2x}+5=0$

24 Solve $9^{x-2}=3^{y}$ and $\log_3 2x=1+\log_3 y$ simultaneously.